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Discussion on the paper by Hird C.C., Pyrah I.C. and Russell D. by Potts D.M. and Ganendra D.

FINITE ELEMENT ANALYSIS OF THE COLLAPSE OF REINFORCED EMBANKMENTS ON SOFT GROUND

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DISCUSSION

Finite element analysis of the collapse of reinforced embankments on soft ground

C. C. HIRD, I. C. PYRAH and D. RUSSELL (1990). *Géotechnique* **40**, No. 4, 633–640

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The Authors show agreement between failure loads predicted by numerical analyses of footings on undrained clay using the finite element program CRISP84 and those predicted by closed-form plasticity solutions. This agreement is also shown in one analysis where modified Cam clay is used as the constitutive model for the soil. We have performed similar work, which indicates that such agreement will not always be found. Thus care must be exercised in applying this program to real engineering stability problems.

Our first concern is the compliance of the numerical solution with the soil model specified. We understand that CRISP employs a simple tangent stiffness approach to solve the non-linear finite element equations. In this approach the applied loads and/or displacements are divided into a number of increments; for each increment the stiffness matrix of the system is based on the stress state at the beginning of the increment and is assumed to be constant. No iteration is performed to ensure that the stress state at the end of the increment is consistent with the constitutive model, and errors can accumulate as an analysis advances. Numerical errors may be reduced by an increase in the number of increments and a reduction in their size, but it is our experience that, when sophisticated constitutive models are used, this works only if many increments are employed (typically more than a thousand if the modified Cam clay model is used). Even this does not guarantee agreement with closed-form solutions for some problems. As the stiffness matrix is based on the stress state at the beginning of the increment and is assumed constant throughout the increment it is difficult to establish when an integration point changes from elastic to plastic behaviour, or from loading to unloading. These difficulties lead to errors in the solution which cannot necessarily be corrected by the use of smaller increments. These difficulties make it unwise to accept the results of analyses using this approach without a check to ensure that the solution is independent of increment size and that the predicted stress state complies with the soil model adopted.

A second area of difficulty arises specifically from the way the modified Cam clay model has been implemented in CRISP. In particular, the strength of the soil changes according to the three-dimensional stress field generated by the solution. CRISP assumes that the slope of the critical state line in the deviator–mean stress space is given by the gradient M. This gradient is independent of the intermediate stress σ_2' and therefore the Lode angle θ where

$$\theta = \arctan\left[(2b-1)/\sqrt{3}\right]$$

$$b = \frac{\sigma_2' - \sigma_3'}{\sigma_1' - \sigma_3'} \tag{1}$$

This results in the angle of shearing resistance ϕ being a function of the Lode angle

$$\phi = \arcsin \frac{(M/\sqrt{3})\cos\theta}{1-(M/3)\sin\theta} \tag{2}$$

For triaxial compression and extension $\theta = -30°$ and $+30°$ respectively and, because the model assumes associated plasticity, $\theta = 0°$ for plane strain failure (Potts & Gens, 1984). Consequently if $\phi = 24°$ in triaxial compression the model gives $\phi = 33°$ in plane strain and $\phi = 34°$ in triaxial extension. This is unrealistic and clearly any user of the program must exercise care when choosing input parameters.

A further difficulty arises because the model does not readily allow a linear undrained strength with depth profile to be adopted. The undrained shear strength c_u which can be derived from the model can be written as

$$c_u = \sigma_{vi}' \frac{M}{\sqrt{3}} \cos\theta \frac{1+2K_0^{NC}}{6} \mathrm{OCR}\,(1+A^2) \left[\frac{2(1+2K_0^{OC})}{(1+2K_0^{NC})(\mathrm{OCR})(1+A^2)}\right]^{\kappa/\lambda} \tag{3}$$

where

$$A = \frac{\sqrt{3}(1-K_0^{NC})}{(M/\sqrt{3})(1+2K_0^{NC})}$$

K_0^{NC} is the coefficient of earth pressure at rest for normally consolidated soil, K_0^{OC} is the coefficient of earth pressure at rest for over-consolidated soil, OCR is the overconsolidation ratio, κ is the slope of the swelling line, λ is the slope of the virgin consolidation line and σ_{vi}' is the initial vertical effective stress in the ground.

It can be seen that the value of c_u varies with Lode angle and hence also with the intermediate principal stress. The ratio of undrained strength c_u in triaxial compression and plane strain is $c_u^{TC}/c_u^{PS} = 0{\cdot}87$.

Considering the footing problem analysed in the Paper, we understand that the initial soil conditions were based on the assumption that an initially normally consolidated soil layer with the water table at the ground surface was modified by the water table being lowered by 2 m and then returned to the ground surface. Assuming that the soil has a saturated bulk unit weight of 18 kN/m^2, this results in a distribution of OCR with depth z below the ground surface of

$$\text{OCR} = 1 + 2{\cdot}40/z \tag{4}$$

Adopting $K_0^{NC} = 1 - \phi^{TC}$ and

$$K_0^{OC} = \text{OCR} * K_0^{NC} - \frac{\mu}{1-\mu}(\text{OCR} - 1) \tag{5}$$

where $\mu = 0{\cdot}3$ as used by the Authors, the c_u profile for the given soil properties becomes

$$c_u = (2{\cdot}30z + 5{\cdot}50)\left(\frac{4{\cdot}29z + 1{\cdot}37}{2{\cdot}91z + 6{\cdot}98}\right)^{0{\cdot}28} \tag{6}$$

Clearly, this is not linear and is strongly nonlinear close to the soil surface where the strength has a considerable effect on the ultimate capacity of the footing. At the soil surface the value of c_u shown in Fig. 11 is calculated to be about 22% higher than that given by equation (6). This reduces to 8% and 6% at depths of 0·5 m and 2 m respectively. The Authors' comparison of their modified Cam clay analysis, which does not have a linear c_u profile, with that of a theoretical solution, which does, is therefore uncertain.

We wonder if this problem of strength specification is understood by all users of programs involving this model, and whether or not some analyses applied in geotechnical engineering may have used strengths rather different from those intended, without this being apparent. It seems desirable to ask the program to plot the actual strengths adopted in the analysis.

We have made comparisons similar to those of the Authors and in particular we have compared results of analyses based on the tangent stiffness approach with those using the standard solution algorithm employed in our finite element program ICFEP. This uses a more sophisticated solution algorithm, based on a modified Newton–Raphson technique which employs a stress point algorithm using an explicit substepping technique with error control (Sloan, 1987). In essence this solution approach iterates for each load increment to ensure that the stress state is consistent with the constitutive model throughout the analysis.

Table 1. **Material properties for the modified Cam clay model**

Overconsolidation ratio	1·0
Specific volume at unit pressure v_1	1·788
Slope of virgin consolidation line λ in $e - \ln p'$ space	0·066
Slope of swelling line κ in $e - \ln p'$ space	0·0077
Slope of critical state line in q–p' plane M	1·2
Elastic shear modulus G/preconsolidation pressure p_c'	100

We have carried out finite element analyses of a smooth footing on a layer of modified Cam clay. A linear c_u profile with depth was obtained by assuming the clay to be normally consolidated with the water level at the ground surface. The soil properties employed are given in Table 1, and Fig. 13 shows the geometry used and the load–displacement curves obtained using both a tangent stiffness approach and the more sophisticated ICFEP approach. The number of increments associated with each curve in this figure indicates the number of increments of displacement used to reach a vertical displacement of 25 mm. Fig. 13 indicates that there is a strong dependency on increment size for tangent stiffness predictions with the ultimate load varying from 7·5 kN to 2·8 kN as the increment size is reduced. With the larger increment sizes there is also a tendency for the load–displacement curve to continue to rise and not to reach a well-defined ultimate failure load. In comparison, the ICFEP predictions are insensitive to increment size and show a well-defined ultimate load. Significantly, the CPU time required for the tangent stiffness prediction with a displacement step size of 0·025 mm was more than seven times that used by the ICFEP analysis with a displacement step of 2·5 mm. Both these predictions are of approximately the same accuracy.

The analytical collapse load for this problem is 1·9 kN and it can be seen that all predictions exceed this value. This occurs because the failure zone is very localized near the soil surface. Reducing the thickness of the elements in the vicinity of the footing from 0·1 m to 0·03 m results in an ICFEP prediction of 2·1 kN for the ultimate load. Clearly, if the element size is further refined the true analytical solution will be

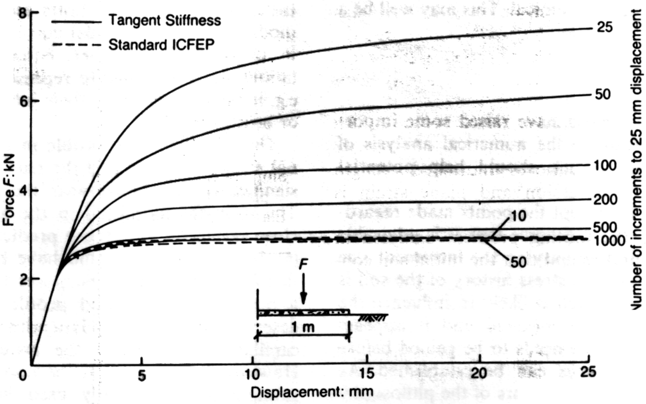

Fig. 13. Coarse mesh foundation analyses: load displacement curves

recovered. For this refined mesh additional problems occurred with the tangent stiffness predictions and it was found that even smaller displacement increments were required to obtain an accurate solution. This particular problem, where the strength is zero at the surface, provides a stringent test for any finite element program; the influence of element size is a finite element discretization problem and not necessarily program-dependent.

We have also reanalysed the footing problem presented by the Authors using the soil properties given. These parameters result in a non-linear distribution of c_u with depth and therefore it is difficult to compare the results with theoretical solutions. However, by comparing the tangent stiffness analyses with those using ICFEP we found that 600–800 increments are needed to obtain a tangent stiffness solution which is in agreement with solutions using ICFEP. This implies that the Authors' analysis with 600 increments is accurate. In this problem the soil immediately under the footing has an OCR of 3–4. When loaded undrained, this material will behave elastically until yield occurs at a point on the state boundary surface near to the critical state. Consequently there will not be large variations in the elasto-plastic stiffness before failure occurs. For the analyses shown in Fig. 13 the soil was assumed to be isotropically normally consolidated and therefore the stress paths had to travel around the state boundary surface to reach failure, resulting in a large variation of the elasto-plastic stiffness. It is our experience that as the non-linearity increases the tangent stiffness approach requires smaller increments. To investigate this further we repeated the Authors' analysis but with slightly different initial soil conditions. Again we started with a normally consolidated layer of clay with the water table at the ground surface. The water table was then lowered and kept 2 m below the soil surface. The soil above the water table was assumed saturated and able to sustain the negative pore pressures generated. This initial condition results in a normally consolidated soil layer with a finite strength at the soil surface which increases linearly with depth. Analyses with this initial soil condition, using the tangent stiffness approach, required more increments to obtain an accurate solution and therefore confirm the above hypothesis.

The results of this comparison show clearly the sensitivity of the tangent stiffness approach to increment size. Unless many increments are used inaccurate predictions, which usually over-predict collapse loads, are likely to be obtained. For boundary value problems which have no analytical solution (the very problems that require a finite element analysis) it is clearly necessary to perform several analyses with different increment sizes to ensure that a correct solution has been obtained.

This has serious implications for users of CRISP and extreme care is required in applying the program to analyse real engineering problems. The Authors' rather general conclusions may therefore be misleading. More sophisticated solution techniques are available which allow the user to have direct control over the solution accuracy and provide accurate solutions which are not as sensitive to increment size. In many situations these solution techniques require less computer resources than those based on a tangent stiffness approach for the same accuracy of prediction. However, as a tangent stiffness approach often gives an answer, even if few increments are used, they could appear economical. This may well be a false economy.

Authors' reply

Potts and Ganendra have raised some important issues relating to the numerical analysis of undrained failure which should help potential users of CRISP.

In principle, we accept the points made regarding increment size, and agree that it is advisable to check the effect of varying the increment size when using CRISP. The stress history of the soil is indeed one factor which is likely to influence the number of increments required and it appears that further experience needs to be gained before any general guide-lines can be established. As inheritors rather than initiators of the philosophy implemented in CRISP, we do not wish to discuss the efficiency of CRISP relative to that of another program, although a more sophisticated algorithm may cut costs.

Regarding the difficulty with the approach adopted in CRISP for calculating the undrained (critical state) strength under generalized stress regimes, it is true that with CRISP it is not currently possible to model simultaneously, yet realistically, the strengths attained under the full range of stress conditions and that care must be taken when specifying input parameters. For a plane strain analysis, our usual approach is to manipulate the parameter M to give a realistic ratio of c_u/σ_{vi}' for the normally consolidated soil in plane strain (in the test problem analysed in the Technical Note the value of M was arbitrary and rather high). The simplest way to verify that the strengths being calculated by the program during the analysis are the same as those intended is to check that in failing elements (i.e. elements close to a critical state) the maximum shear stress is equal to the desired undrained shear strength. A preliminary analysis, performed by the Authors, consists of subjecting a rectangular vertical slice of the mesh to a strain-controlled undrained compression test by allowing the vertical side boundaries to separate uniformly until every element has failed. The variation of the maximum shear stress with depth is plotted and compared with the theoretical strength profile. It should perhaps be remembered that, apart from the difficulty raised by Potts and Ganendra, the modified Cam clay model currently implemented in CRISP possesses other, equally serious limitations when it is used to represent natural clays, e.g. it cannot reproduce their inherent anisotropy or brittleness.

The linear strength profile in Fig. 11 is indeed not strictly accurate and the true profile becomes significantly non-linear near the ground surface. The strengths mobilized in the analysis were in close agreement with those predicted by equation (6). In retrospect, it would have been more sensible for benchmarking purposes to have adopted a normally consolidated profile in the manner described by Potts and Ganendra (i.e. by allowing capillary action above the groundwater level). However, it appears that the non-linearity of the strength profile actually used has a negligible effect on the solution. Our results show that slip occurred on the interface beneath the loaded area (at a shear stress of $4{\cdot}25$ kN/m^2) even though the shear strength was lower at certain integration points in the neighbouring soil elements. This implies that yielding was constrained in those elements to some degree and is a manifestation of the discretization problem referred to by Potts and Ganendra. It has to be appreciated that, according to plasticity theory, a displacement discontinuity develops at the underside of the loaded area. Interface elements provide a convenient means of modelling this discontinuity and avoiding an overprediction of the collapse load. For certain footing problems, it has been shown by Van Langen & Vermeer (1991) that improved results can be obtained by placing interface elements, not only at the base of the footing, but also in the soil near the singularities of displacement at the edges of the footing.

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